

37. Compute a hypergeometric closed form of the sum  $s_n = \sum_{k=0}^n \frac{4}{(2k-1)(2k+1)}$  by applying Gosper's algorithm.
38. Show that the harmonic numbers  $(H_n)_{n \geq 0}$  are not hypergeometric using Gosper's algorithm.
39. For which values of  $\alpha$  is  $a_k = \binom{2k}{k} \alpha^k$  Gosper summable? Compute  $\sum_{k=0}^n a_k$  for the admissible values of  $\alpha$ .
40. What hypergeometric series  $F$  satisfies  $zF'(z) + F(z) = \frac{1}{1-z}$ ?