33. Let $(f_n)_{n\geq 0}$ and $(g_n)_{n\geq 0}$ be sequences. Prove that the difference operator satisfies the product rule
$$\Delta_n(f_n g_n) = (\Delta_n f_n) g_n + f_{n+1}(\Delta g_n), \qquad n \geq 0.$$
Use this law to deduce the formula for summation by parts,
$$\sum_{k=0}^{n} f_k(\Delta_k g_k) = f_{n+1}g_{n+1} - f_0 g_0 - \sum_{k=0}^{n}(\Delta_k f_k) g_{k+1}.$$

34. Let $(a_n)_{n\geq 0}$ be the coefficient sequence of an algebraic power series. Show that then the sequence of partial sums $\left(\sum_{k=0}^{n} a_k\right)_{n\geq 0}$ is the coefficient sequence of an algebraic power series.

35. Let T_n be the number of tilings of a $3 \times n$ rectangle with straight trominoes (i.e., 1×3 and 3×1 pieces).
    (a) Determine a recurrence relation for T_n.
    (b) Express the partial sum $s_n = \sum_{k=0}^{n} T_k$ in terms of T_n.

36. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\left(a_n\right)_{n\geq 0}$ is given by the recurrence
$$a_{n+3} = 5a_{n+1} - 4a_n, \qquad a_0 = a_1 = 1, \quad a_2 = 2.$$