25. Given n people numbered from 1 to n sitting at a round table. Starting from person 1 in clockwise order every second person leaves until only one person remains (the first person to leave is person 2). Let $J(n)$ denote the number of the remaining person. Determine $J(n)$.

26. Show that a sequence $\big(a_n\big)_{n\geq 0}$ is holonomic if and only if there exist polynomials $p_0, \ldots, p_r \in \mathbb{K}[x]$ and $q \in \mathbb{K}[x]$ such that

$$p_r(n)a_{n+r} + \cdots + p_1(n)a_{n+1} + p_0(n)a_n = q(n), \qquad n \in \mathbb{N}.$$

27. Derive the second order linear differential equation satisfied by the Gauss hypergeometric function

$${}_2F_1\left(\begin{matrix} a, b \\ c \end{matrix} \middle| z\right) = \sum_{n\geq 0} \frac{(a)_n (b)_n}{(c)_n n!} z^n = \sum_{n\geq 0} \alpha_n z^n$$

by computing the first order recurrence satisfied by the coefficient sequence $(\alpha_n)_{n\geq 0}$ and transforming the recurrence into a differential equation satisfied by the generating function.

28. Let $C(x) = \sum_{n\geq 0} C_n x^n$ be the generating function of the Catalan numbers. In the lecture we deduced from the equation $xC(x)^2 - C(x) + 1 = 0$ the explicit representation

$$C(x) = \frac{1 - \sqrt{1-4x}}{2x}.$$

   (a) What rules out the possibility $C(x) = \frac{1+\sqrt{1-4x}}{2x}$?

   (b) Prove or disprove that the multiplicative inverse of $C(x)$ is holonomic.