

9. Prove Theorem 18: Show that for the formal derivation and integration as defined in the lecture and for all $a(x) \in \mathbb{K}[[x]]$ it holds that

$$(1) D_x \int_x a(x) = a(x)$$

$$(2) \int_x D_x a(x) = a(x) - a(0)$$

$$(3) [x^n]a(x) = \frac{1}{n!} (D_x^n a(x)) \Big|_{x=0}$$

10. Let the Euler operator be defined as $\theta_x = xD_x$. Show that

$$\theta_x(\theta_x - 1) \cdots (\theta_x - k + 1)f(x) = x^k f^{(k)}(x).$$

11. Prove Theorem 19 (Multiplicative Inverse):

Let $a(x) \in \mathbb{K}[[x]]$. Then there exists a series $b(x) \in \mathbb{K}[[x]]$ with $a(x)b(x) = 1$ if and only if $a(0) \neq 0$.

12. Let $(a_n(x))_{n \geq 0}, (b_n(x))_{n \geq 0}$ be convergent sequences of formal power series with respective limits $a(x), b(x) \in \mathbb{K}[[x]]$.

Show that then also $(c_n(x))_{n \geq 0}$ with $c_n(x) = a_n(x) + b_n(x)$ is a convergent sequence of formal power series with limit $a(x) + b(x)$.