MATCONT: A Matlab package for the numerical study of dynamical systems

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Software name: MATCONT

Short description: MATCONT is a graphical Matlab software package for the interactive numerical study of parametrized ODEs. Matlab provides dynamic simulation (all Matlab solvers are incorporated) and the numerical continuation of curves of equilibria, limit points, Hopf points, limit cycles, fold, flip and torus bifurcations of limit cycles, branch points of equilibria and periodic orbits.

It can detect and compute bifurcation points, perform branch switching at Hopf points, branch points and period doubling points.MATCONT uses the Matlab symbolic toolbox whenever it is available; higher order derivatives are important e.g. in the computation of normal forms.

The sparsity of the discretized systems for the computation of limit cycles and their bifurcation points is exploited by using the standard Matlab sparse matrix methods.

Among the recent additions to MATCONT we illustrate the computation of normal form coefficients for bifurcations of limit cycles.

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Abstract

We consider generic parameterized autonomous ODEs of the form $\frac{dx}{dt} \equiv \dot{x} = f(x,\alpha)$, where $x \in \mathbb{R}^n$ is the vector of *state variables*, $\alpha \in \mathbb{R}^m$ represents *parameters*, and $f(x,\alpha) \in \mathbb{R}^n$. There are several interactive software packages for analysis of dynamical systems defined by ODEs. The most widely used are AUTO86/97[1], CONTENT[4] and XPPAUT.

The Matlab software package MATCONT provides an interactive environment for the continuation and normal form analysis of dynamical systems. This analysis is complementary to the simulation of the systems and can be used in their identification, control, and optimization. MATCONT is designed

to exploit the power of Matlab. It is developed in parallel with the continuation toolbox CL_MATCONT, a package of Matlab routines that can be used from the command line.

We consider the following model of a feedback control system, described in [2], [3]:

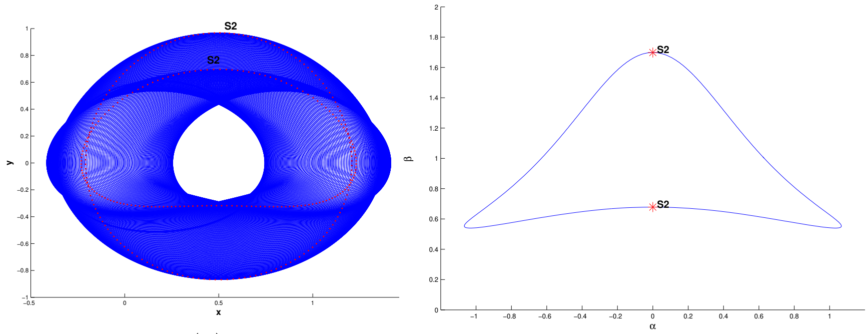
$$\begin{cases} \dot{x} & = & y \\ \dot{y} & = & z \\ \dot{z} & = & -\alpha z - \beta y - x + x^2 \end{cases} \tag{1}$$

We compute numerically a family of periodic orbits starting from the Hopf point for $\alpha = 1$ and $\beta = 1$ at $(0,0,0)$. We choose $N = 20$ test intervals and $m = 4$ collocation points for the discretization. We detect two period doubling points with period 6.36407 at $\alpha = \pm 0.6303020$, respectively. The multipliers in the first PD point are inside the unit circle, so the periodic orbit is stable. In the second PD point, there is one multiplier $<< -1$ therefore the periodic orbit is unstable. It is know that the periodic normal form at the Period Doubling (PD) bifurcation is presented by

$$\begin{cases} \frac{d\tau}{dt} & = & 1 + a\xi^2 + \cdots \\ \frac{d\xi}{dt} & = & c\xi^3 + \cdots, \end{cases} \tag{2}$$

where $\tau \in [0, 2T]$, T is the period of the cycle, ξ is a real coordinate on the center manifold that is transverse to the limit cycle, $a, c \in \mathbb{R}$ and dots denote nonautonomous $2T$-periodic $O(\xi^4)$-terms.

In the PD points the normal form coefficient c is computed. In the first PD point $c = -4.267737e - 002$. Therefore the periodic orbit with doubled period in the first PD point is stable ($c < 0$). This was confirmed by computing the periodic orbit and monitoring the multipliers. In the second PD point the normal form coefficient $c = 4.268605e - 002$. Hence the periodic orbit with doubled period is unstable ($c > 0$). By computing the orbit with doubled period and monitoring the multipliers in this second PD point we found that this orbit is indeed unstable; two multipliers are not inside the unit circle. From the first PD point we compute the branch of PD-cycles. We monitor two test functions, namely c and a test function for the strong 1:2 resonance (S2). We detected two strong 1:2 resonance on this curve for $\alpha = 0$. By monitoring the multipliers in those S2 points, we see that in both cases there are two multipliers equal to -1. This family is represented in Fig. 1.



(a) (b)

Figure 1: (a)PD curve, with two S2 points for $\alpha = 0$,(b)PD curve in (α, β)-space.

References

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